1a) A\* uses a priority queue as its open list. The priority queue is based on the f value of nodes such that the node with the minimum f value gets expanded first. The f value is in turn dependant on the g value and h value (f=g+h) where h is the heuristic function and g is the distance travelled from the start node till now. Every time the agent moves 1 step the g value will increase by 1. The h value on the other hand is based on the Manhattan distance between the current node and the goal. In this example, the g value of each of the nodes a, b, c when it enters the open list is equal to 1. At the same time the h value for node a, c is 4 whereas for node b(towards the east) it is 2. Hence the node b has the minimum h value and with the g value remaining the same for all three nodes, the f value for node b is the minimum. So the node that will expand first is Node b. Hence the agent moves to east initially.



Node b: H=2; G=1; F=3

Node c: H=4; G=1; F=5

Node a: H=4; G=1; F=5

1b) A\* always expands nodes with the lowest f value. It terminates when it expands the goal node G with cost c\*, which is the optimal cost. Therefore, it does not expand nodes with f value greater than c\*. Moreover it expands all nodes with f value less than c\* because it will expand G only after expanding all nodes with f value less than c\*. This is true unless there is an infinite state space. Hence A\* will always reach a target in a finite graph or inform if there is no path.   
A\* only expands nodes that have lower cost than the optimal cost. Hence it expand all n such that f(n) < c\*. The optimal cost c\* is bounded by m^2 in a gridworld where m is the number of unblocked nodes. Hence the number of moves that the agent makes before reaching the target is bounded by the square of the number of unblocked cells.

4. A heuristic is consistent if the heuristic function follow the triangular inequality h(n) <= c(n, a, n') + h(n'). Manhattan distance always gives the shortest path between two nodes in a grid. H(n) is the shortest distance between the node n and the Goal node. c(n, a, n') + h(n’) along any ‘a’ will either be equal to or greater than h(n) depending upon the amount of deviation due to ‘a’ from the Manhattan path of n. Hence c(n, a, n') + h(n’) will either be greater than or equal to h(n) and the triangular inequality will hold.

The Heuristic of adaptive A\* algorithm is the goal distance gd(n) of a node n. Also, gd(n) <= gd(n’) + c(n, a, n’). This is because any c(n, a, n’) is adding an incremental cost towards the goal. Now if c(n, a, n’) increases then also gd(n) <= gd(n’) + c(n, a, n’) will hold true. Therefore the gd(n) is a consistent heuristic even if the action cost increases.